

On the Pauli principle violation in QFT

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Abstract

We propose a new mechanism for a "small" violation of Pauli Principle in the framework of Quantum Field Theory. Instead of modification of algebra - commutation relations for fields - we introduce spontaneous violation of Pauli Principle which is proportional to the vacuum fermionic condensate.

Getting older some of the theorists turn to the foundations of Quantum Mechanics. This is exactly my case. I am going to discuss the possibility of Pauli Principle breaking - one of the cornerstone of Quantum Field Theory (QFT). My talk is based on yet unpublished paper (un)written in collaboration with my old friend Sasha Dolgov (at early stage Maxim Pospelov participated to our discussions).

1 Introduction

The great merit of Pauli Principle is that it can be formulated in terms understandable to any person from the street. On the other hand the proof of Pauli Principle is based on a rather advanced formalism of QFT comprehensible to the tiny fraction of experts. Feynman wrote in his famous Lectures on Physics: “It appears to be one of the few places in physics where there is a rule which can be stated very simply, but for which no one has found a simple and easy explanation. The explanation is deep down in relativistic quantum mechanics. This probably means that we do not have a complete understanding of the fundamental principle involved.” [1].

To understand principle better sometime it is useful to break it. I got involved in this business when I have seen the paper by Dolgov and Smirnov [2], where the authors proposed a fractional statistic for neutrino. (Fractional statistics for charged particles such as electron is excluded by experiment). In this way they wanted to get a Bose condensation of neutrino in vacuum and explain the origin of dark matter. My recollection was that it is absolutely impossible to get Pauli Principle violation in the framework of QFT and that it is well known in literature that it is impossible. I was wrong - it is possible and there is vast literature on this subject.

2 Short History of Discovery and Long History of Breaking

Exclusion Principle was introduced by Pauli in 1925 [3]. The first formal proof in QFT was developed by him fifteen years later [4]. The long list of papers that improved and purified the original proof can be found in any

book on axiomatic field theory. The best reference, as I know, is still the book of R.F.Streater and A.S.Wightman [5].

Non-standard types of statistics such as parastatistics had been known for a long time [6]. It was so to say "large" violation of Pauli Principle. Near a dozen or papers were published on this subject till 1987. In 1987 the first model for small violation of Pauli Principle was constructed in the framework of QM [7]. In this model the bilinear commutation relations for annihilation and creation operators were modified to trilinear relations with small parameter. It was a great success. But it was found immediately that generalization of bilinear relations to trilinear ones is rather difficult procedure in QFT. This modification inevitably leads to some pathology and consistent QFT with fractional statistics does not exist [8]. After this work near a hundred of papers with different modifications of Algebra of Operators were published. In a very recent paper [9] a kind of "No-Go Theorem" was proven. It was found that it is difficult (if possible) to get a small violaiion of Pauli Principle in QFT with bilinear algebra.¹

We suggest not to modify Algebra of Fields and not to destroy Pauli Principle by brute force. Instead of genuine breaking we suggest to imitate Pauli Principle violation exactly like "Spontaneous Symmetry Breaking" imitates symmetry violation.

3 Simulation of Symmetry Breaking

The phenomenon of Spontaneous Symmetry Breaking is getting known to students from a course on general physics when they learn about ferromagnetic material. Suppose that we have a piece of iron. Electromagnetic interaction of electrons in metal is $O(3)$ invariant. Suppose now that we switch-on an external electric current \mathbf{j} . Electric current produces magnetic field \mathbf{B} . It is clear that interaction of non-zero external current with electrons in metal breaks $O(3)$ invariance. If we switch-off the current the $O(3)$ symmetry will be restored. On the other hand due to magnetization of ferromagnet the external magnetic field \mathbf{B} still remains non-zero. As a result we get $O(3)$ invariant system that interacts with $O(3)$ non-invariant external field. This effect has a name "Spontaneous Symmetry Breaking" though symmetry is not broken. We get a sort of imitation of $O(3)$ symmetry violation.

¹This paper contains rather complete list of references on the Pauli Principle Violation.

The standard description of Spontaneous Symmetry Breaking in QFT step by step follows that example. First one considers a system with internal symmetry G and Lagrangian $L_{SYM}(\phi)$, where ϕ is a general symbol for fields. We suppose that fields ϕ are transformed in some nontrivial way under symmetry transformation G . At second step one switches-on an external classical source (current) J for field. In this case the total Lagrangian is

$$L = L_{SYM} + J\phi, \quad (1)$$

and external source breaks the symmetry. If the current is nonzero, i.e. $\langle J \rangle \neq 0$, it produces nonzero classical field in vacuum, i.e.

$$\langle \phi \rangle_J = \phi_{cl} \quad (2)$$

Fluctuations near this classical fields are described by ϕ_{qu}

$$\phi_{qu} = \phi - \phi_{cl}. \quad (3)$$

They interact with non-invariant object - external classical field.

At last step one switches-off the current, i.e. one puts $J \equiv 0$. In this case the term that breaks the symmetry goes to zero $J\phi = 0$. It happens that for some systems the equation of motion for v.e.v. of the field ϕ is non-zero even for zero current $J = 0$, i.e.

$$\langle \phi \rangle_0 = \phi_{cl} \neq 0. \quad (4)$$

That is non-zero vacuum condensate.

Symmetric interaction of fluctuations with non-symmetric condensate produces imitation of symmetry breaking. This is Nambu-Goldstone mechanism of spontaneous symmetry breaking.

3.1 Scalar Condensate

In QFT there exists a unique explicit example of self-interaction that produces a condensation of scalar fields. It is enough to take a special potential for scalars

$$V(\phi) = \lambda[\phi^2 - \phi_{cl}^2]^2. \quad (5)$$

In the Standard model we use similar potential for Higgs fields to produce vacuum condensate. As a result propagating $SU(2)$ and $U(1)$ massless gauge bosons and massless Goldstone bosons scatter on this vacuum condensate. There is also inelastic scattering on the condensate that mixes vector gauge bosons with Goldstone scalar bosons. The net effect is that instead of massless gauge and Goldstone bosons one gets massive gauge bosons

3.2 Vector Condensate

There exist a vast literature on a Lorentz symmetry breaking. Fortunately in the case of QED any "reasonable" violation of Lorentz symmetry leads to a very simple modification of the Standard Lagrangian L :

$$\delta L = g\epsilon_{\mu\nu\alpha\beta}n_\mu A_\nu F_{\alpha\beta} \quad (6)$$

where A_ν is four-potential for e.m. field, $F_{\alpha\beta}$ is e.m. field-strength tensor, and numerical vector n_μ breaks Lorentz symmetry. There are two schools of thinking: one treats n_μ as an external vector, and the second one treats it as a vacuum condensate of some vector field B_μ , i.e. $n_\mu \equiv \langle B_\mu \rangle_0$. The explicit mechanism for vector field condensation in vacuum does not exist in 4D QFT. Nowadays it is not a great disaster. One can consider our space-time as a 4D brane in multi-dimensional world. Vector field B_μ can be a zero mode living on this brane.

In any case light propagates through vector condensate and has different refraction indexes for left- and right-polarized photons. That is an explicit violation of Lorentz symmetry and CPT symmetry.

3.3 Fermion condensate

Let us introduce a source for fermions and change an initial Lagrangian:

$$L \longrightarrow L + \bar{J}\psi + \bar{\psi}J, \quad (7)$$

where J, \bar{J} are "classical" currents for fermions, i.e. some grassmanian numbers. The introduction of a source term for fermions is rather standard trick in QFT. In this way one can construct partition function $Z(J, \bar{J})$ and generate the complete set of fermionic correlators $\langle \psi_1 \dots \bar{\psi}_n \rangle$ as a variational derivatives of $Z(J, \bar{J})$ over currents J, \bar{J} at $J = 0$.

In the case of $J \neq 0$ the nonzero current generates nonzero expectation value of the fermionic field:

$$\langle \psi \rangle_J = \xi \neq 0, \quad (8)$$

where ξ is also a grassmanian number. Nonzero value of ξ violates Lorentz and rotational symmetry. It is interesting to understand whether it possible to have nonzero value of ξ at zero current $J = 0$. Actual mechanism for spontaneous breaking of symmetry, i.e. for vacuum condensation of ξ , is unknown. In the standard QFT in four dimension it is impossible. But we can think about our space-time as a four-dimensional brane in multi-dimensional space with fermionic zero mode living on the brane. Another possibility is that our Lord just forgot to switch-off the fermionic current J .

In any case we will assume that ξ is nonzero. Certainly it violates Lorentz symmetry. In addition it should violate the Pauli Principle. Indeed consider a simple QFT model.

$$L = \bar{\psi}(\hat{p} - m)\psi + 1/2\phi(\hat{p}^2 - m^2)\phi + \lambda\phi(\bar{\psi}\psi) + \bar{J}\psi + \bar{\psi}J \quad (9)$$

where $\phi(x)$ and $\psi(x)$ are neutral boson and fermion fields. Equations of motion looks like

$$(\hat{p} - m)\psi + \lambda\phi\psi + J = 0; \quad (\hat{p}^2 - m^2)\phi + \lambda(\bar{\psi}\psi) = 0. \quad (10)$$

For classical nonzero constant current

$$J(x) \equiv J = m\xi \neq 0 \quad (11)$$

we get that

$$\langle \psi \rangle_J \equiv \xi, \quad (12)$$

$$\langle \phi \rangle_J \equiv \frac{\lambda}{m^2}\bar{\xi}\xi \quad (13)$$

The propagation of the excitations in the vacuum with these two condensates

$$\psi = \xi + \psi_q; \quad \phi = \frac{\lambda}{m^2}\bar{\xi}\xi + \phi_q, \quad (14)$$

is described by the quadratic form

$$L^{(2)} = \bar{\psi}_q(\hat{p} - \bar{m})\psi_q + \frac{1}{2}\phi_q(\hat{p}^2 - m^2)\phi_q + \lambda\phi_q[\bar{\xi}\psi_q + \bar{\psi}_q\xi], \quad (15)$$

where $\bar{m} = m - \frac{\lambda^2}{m^2} \bar{\xi} \xi$. It is clear that the last term proportional to λ describes inelastic scattering on the fermionic condensate that transforms fermions into bosons and visa verse:

$$Bosons \iff Fermions \quad (16)$$

Evidently such transformation breaks statistic. The direct way to calculate statistic of the excitations that corresponds to ψ_q and ϕ_q is to diagonalize this quadratic form. Technically this rather tricky problem. The proper way to diagonalize it is to quantize a system in a box. In this way we reduce QFT problem to QM problem of the one given level.

4 QM model

Consider field operators $\phi(x)$ and $\psi(x)$ as an expansion over plane waves in the box

$$\phi(x) = \sum_p \frac{1}{\sqrt{2\omega(p)}} [a(p) \exp(ipx) + a^\dagger(p) \exp(-ipx)], \quad (17)$$

and

$$\psi(x) = \sum_p \frac{1}{\sqrt{2\omega(p)}} [b(p) u(p) \exp(ipx) + h.c.], \quad (18)$$

where $\omega^2 = \mathbf{p}^2 + m^2$, and $(a(p), a^\dagger(p))$ and $(b(p), b^\dagger(p))$ are annihilation and creation operators for the original scalar and spinor fields. For the mode with given 3-momenta \mathbf{p} we have a system with two degrees of freedom, i.e. simple Quantum Mechanics

$$H = \omega(p) [aa^\dagger + bb^\dagger] + \lambda [a^\dagger \zeta^\dagger b + b^\dagger \zeta a], \quad (19)$$

with grasmanian parameter $\zeta = (\bar{u}\xi)/2\omega$ and creation and annihilation operators that satisfy well-known algebra

$$[a, a^\dagger]_- = [b, b^\dagger]_+ = 1; \quad [a, a]_- = [b, b]_+ = [a, b]_- = [a, b^\dagger]_- = 0. \quad (20)$$

One can verify that this algebra is invariant under one-parameter group of invariance $(a, b) \rightarrow (A, B)$:

$$a = [1 - \frac{1}{2}(\beta^* \beta)(\zeta^* \zeta)] A + \beta(\zeta^* B) \quad (21)$$

$$b = -\beta^* A \zeta + [1 + \frac{1}{2}(\beta^* \beta)(\zeta^* \zeta)] B, \quad (22)$$

where β is an arbitrary complex number.

Operators A, B satisfy the same algebra

$$[A, A^+]_- = [B, B^+]_+ = 1; \quad [A, A]_- = [B, B]_+ = [A, B]_- = [A, B^+]_- = 0. \quad (23)$$

To diagonalize quadratic Hamiltonian we should take

$$\beta = \beta^* = -\lambda/2\omega(p). \quad (24)$$

In the new canonical coordinates Hamiltonian looks like a sum of bosonic oscillator and fermionic oscillator:

$$H = \omega_1 A A^+ + \omega_2 B B^+ \quad (25)$$

with

$$\omega_1 = \omega + \frac{\lambda^2 m}{16\omega^3} \bar{\xi} \xi, \quad (26)$$

$$\omega_2 = \omega - \frac{\lambda^2 m}{16\omega^3} \bar{\xi} \xi. \quad (27)$$

Mixing between bosonic subsystem (a, a^+) and fermionic subsystem (b, b^+) leads to a repulsion of the levels (with center of mass constrain)

$$\omega_1 + \omega_2 = 2\omega. \quad (28)$$

5 Statistics

In terms of diagonal variables the spectrum of Hamiltonian is known and one can calculate the average number of particle at given state using the standard rules of Statistical Mechanics. For particles that are created by operator A^+ we get a canonical Bose distribution:

$$\langle N \rangle_{Bose} = \langle A A^+ \rangle = \frac{1}{\exp(\omega_1/T) - 1}. \quad (29)$$

with shifted frequency ω_1 . For particles that are created by operator B^+ we get a canonical Fermi distribution

$$\langle N \rangle_{Fermi} = \langle B B^+ \rangle = \frac{1}{\exp((\omega_2 - \mu)/T) + 1}, \quad (30)$$

where μ is a chemical potential. These are the distributions for the diagonal states.

In terms of the initial particles that are created in collisions the same equations look like a mixed statistic . Indeed if we introduce distributing numbers for initial particles

$$\langle n \rangle_B = \langle aa^+ \rangle, \quad (31)$$

and

$$\langle n \rangle_F = \langle bb^+ \rangle, \quad (32)$$

we get

$$\langle n \rangle_F = (1 + \beta^2 \zeta^+ \zeta) N_{Fermi} - \beta^2 \zeta^+ \zeta N_{Bose}, \quad (33)$$

and

$$\langle n \rangle_B = (1 - \beta^2 \zeta^+ \zeta) N_{Bose} + \beta^2 \zeta^+ \zeta N_{Fermi}, \quad (34)$$

where $\beta = -\lambda/2\omega$,

As a result for the distribution of initial "neutrinos" we get

$$\langle n \rangle_\nu = [1 + O(\lambda \bar{\xi} \xi)] \frac{1}{\exp((\omega - \mu)/T) + 1} - \frac{\lambda^2 m}{32\omega^4} \bar{\xi} \xi \frac{1}{\exp(\omega/T) - 1}, \quad (35)$$

i.e. a piece of a standard Fermi distribution with slightly modified frequency plus a fraction of Bose distribution. The admixture of Bose statistic is proportional to the condensation of Fermi field $\bar{\xi} \xi$ and can be made arbitrary small.

6 Back to QFT

In terms of field variables

$$\phi(x) = \sum_p \frac{1}{\sqrt{2\omega(p)}} [a(p) \exp(ipx) + a^+(p) \exp(-ipx)], \quad (36)$$

$$\psi(x) = \sum_p \frac{1}{\sqrt{2\omega(p)}} [b(p) u(p) \exp(ipx) + h.c.] \quad (37)$$

the transformation of operators $(a, b) \Rightarrow (A, B)$ looks like non-local transformations:

$$\phi(x) \Rightarrow [1 - \frac{\lambda^2 m}{64} \bar{\xi} \xi \frac{1}{(-\nabla^2 + m^2)^2}] \phi(x) - \frac{\lambda}{4} \frac{1}{(-\nabla^2 + m^2)} [\xi \bar{\psi} + \psi \bar{\xi}] \quad (38)$$

$$\psi(x) \Rightarrow [1 + \frac{\lambda^2 m}{64} \bar{\xi} \xi \frac{1}{(-\nabla^2 + m^2)^2}] \psi(x) + \frac{\lambda m}{4} \frac{1}{(-\nabla^2 + m^2)} \phi(x) \xi. \quad (39)$$

By construction it is clear that these non-local transformations do not violate causality.

7 Conclusions

Let me summarize the results.

The Fermion vacuum condensate simulates Pauli Principle breaking. This is a new way to play with Pauli Principle Breaking in QFT. We hope that we have made a step in true direction.

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